Fill ‘er Up
(Patterns)

Objective

Students will be able to interpret, predict and sketch graphs of related functions.

Overview of the Lesson

Two containers with different shapes are shown to the class. Each time a measure of water is poured into the container, the number of this measure (x) and the resulting height of the water (y) are plotted on a graph as an ordered pair. Students demonstrate the procedure of filling the measure with water, pouring the water into the container, and measuring the height. Students plot the points and make predictions. Following the same procedure, three more bottles with different shapes are displayed with three completed graphs. Students work in groups to match these containers with their related graph. Each group must be able to supply a rationale for their decision. Keeping the same variables, students are asked to intuitively sketch the graphs of some additional irregular containers.

Materials

1. Two clear containers with different shapes
2. Six additional clear containers with different shapes and large graphs related to three of these containers
3. Container filled with colored water (Rice or sand can be used instead of water)
4. 12-inch ruler
5. Graph paper for students
6. Measuring devices (i.e., cup, ladle, graduated cylinder)
**Procedure**

Begin the lesson by holding up two clear different shaped containers. Ask students to describe these two containers, paying particular attention to similarities and differences between the two. Next, show a measuring cup and ask students to hypothesize how the graphs would look if measures of water were systematically added to each container and the height of the liquid measured. Allow students to respond, and then tell them that they will have the opportunity to experiment to discover if their hypotheses are correct.

Note: You may wish to have individual groups work on the activity or proceed with the whole class as shown in the video.

Explain the procedures of the initial investigation:

- Assign the tasks so that one student is responsible for pouring the measures of colored water, another student carefully measures the height of the water each time a measured amount of water is added, and the third student plots the ordered pairs (number of measures, height of the water) on the graph. This process is continued until the container overflows. Reassign these tasks each time a different container is investigated.

- Discuss how the graph should be set up. It is suggested that the scale for the number of measures be placed on the x-axis and the scale for the height of the liquid be placed on the y-axis.

- As the containers are being filled, have all students plot the points on their individual graphs while a student is plotting points on the large display graph.

- Once the two containers are filled, and the two graphs are completed, ask students to study them and share how they are alike and different. Responses should include such statements as: each time a measure of water is added, the height increases; the height of the liquid increases faster in container A than in container B; the height of the liquid increases slower when the container is wide and faster when the container is narrow.

Discuss the relationship between the size of the container and the steepness of the graphs. For example in a circular container, the narrower the opening, the steeper the rise, and the wider the opening, the more gradual the rise. Also students should notice that if the container is very irregularly shaped, that the graph may have periods of steep and gradual rises.

Once students understand these concepts, show the three completed graphs and three new containers. Have students work in groups to collaboratively determine which graph represents which container. Groups must be able to justify their reasons for their selections. You may wish to perform an experiment to resolve any controversies.
Finally, display some unusually odd-shaped containers. Keeping the same variables, have students sketch the graphs. It is essential that students apply problem solving skills, number sense skills and reasoning skills when predicting and interpreting graphs. Plotting points obtained from systematically filling containers like fishbowls, stretched soda bottles, and vases should make some interesting graphs.

Mathematically Speaking . . .

Depending on the readiness of the students, you may decide to introduce the term “function.” Discuss with the students what a function is and how the sets of data represent functions. A function is a relationship in which there is only one value of the dependent variable for each value of the control (independent variable). In this case, the control variable is the measure and the dependent variable is the height. Therefore, the height of the liquid is controlled or dependent on the number of measures.

Extensions & Connections

Provide groups of students with some irregularly shaped containers and a small measuring device such as a teaspoon. The smaller measuring device will generate more ordered pairs as it will require more “scoops” to fill the container. As more points are plotted, parts of the graph will more closely approximate a curve. Have students sketch the graph of a narrow and wide cylindrical container. If available, you may wish to show this on a graphing calculator or computer.

Resources

Ideas for Online Discussion

(Some ideas may apply to more than one standard of the NCTM Professional Standards for Teaching Mathematics.)

Standard 1: Worthwhile Mathematical Tasks

1. What important mathematical content is incorporated in this lesson?

2. Do you feel this lesson captured students’ curiosity and allowed them to pursue their hunches?

Standard 3: Students’ Role in Discourse

3. “Discourse should be focused on making sense of mathematical ideas.” (p. 45) However sometimes students give answers which don’t seem to “make sense.” How do you resolve this?

Standard 4: Tools for Enhancing Discourse

4. In the video, the teacher did not mention to her class that the relationships which they were investigating were functions. Do you believe that the use of formal terminology should be introduced at this point, or postponed until students are engaged in studying the topic formally?

Standard 6: Analysis of Teaching and Learning

5. Do you consider the final assignment, where students were asked to sketch the graph of the irregularly-shaped Pepsi bottle, to be an appropriate and adequate assessment task for this lesson? Defend your answer. What tasks do you feel would adequately assess students’ understanding?

6. Having seen or taught this lesson, reflect on the appropriateness of the task, use of discourse, and the establishment of a positive learning environment.